"On the Drift produced in Ions by Electromagnetic Disturbances, and a Theory of Radio-activity." By GEORGE W. WALKER, M.A., A.R.C.Sc., Fellow of Trinity College, Cambridge, Lecturer on Physics in the University of Glasgow. Communicated by Professor A. GRAY, F.R.S. Received December 16, 1904,—Read January 26, 1905.

Some time ago I showed* how the equations of motion of a free ion under the influence of a harmonic train of plane waves might be completely integrated, subject to the restriction that the viscous effect of radiation from the ion may be neglected.

The equations are closely analogous to those for a simple pendulum, and by following out the analogy in the case where the pendulum makes complete revolutions, it is easy to show that while the passage of a complete wave restores the initial velocities of the ion, its position in space is altered. This change of position cannot be accounted for entirely by the change due to velocity which the ion may be assumed to possess before the wave reaches it.

The continuance of the waves thus involves the result that the ion must continue to change its position in space. It will thus appear to move in a definite manner which can be determined in terms of the initial circumstances of the ion and the constants of the train of waves. The result is very remarkable, and is not confined to an infinite train of harmonic waves. Similar results follow in the case of any form of electro-magnetic disturbance propagated in a straight line.

I propose here to discuss the case of a plane polarised disturbance propagated in a straight line. Let the electric force be $X = X_0 f(Vt - z)$ where V is the velocity and z the direction of propagation. Associated with this we must have a magnetic force $M = X_0/V\ f(Vt - z)$ in a direction at right angles to that of X. If m be the inertia and e the charge of the ion, the equations of motion may be written

$$m\ddot{x} = e(X - \dot{z}M),$$

$$m\ddot{y} = 0,$$

$$m\ddot{z} = +e\dot{x}M.$$

We may thus confine attention to the motion in the xz plane. We have

$$\ddot{x} = \frac{e}{m}\frac{X_0}{V}(V - \dot{z})f(Vt - z),$$

$$\ddot{z} = \frac{e}{m}\frac{X_0}{V}\dot{x}f(Vt - z).$$

* 'Roy. Soc. Proc.,' vol. 69, p. 394; 'Phil. Mag.,' 1903, vol. 6, p. 537.

If we take a moving origin so that $Vt-z=\zeta$, then ζ will be the distance of the ion from a plane moving with the disturbance, and reckoned positive in the direction from which the waves come. The equations then take the form

$$\ddot{x} = \frac{e}{m}\frac{X_0}{V}\dot{\zeta}f(\zeta),$$

$$\ddot{\zeta} = -\frac{e}{m}\frac{X_0}{V}\dot{x}f(\zeta).$$

Thus in general

$$\dot{x}^2+\dot{\zeta}^2 = \text{const.}$$

is an integral; it indicates that the energy of the ion relative to the moving origin is constant.

We also get

$$\dot{x} = a+\frac{e}{m}\frac{X_0}{V}\int_0^{\zeta} f(\zeta)\,d\zeta,$$

where a is a constant,

We may note that if the disturbance is such that

$$\int_0^{\zeta_1} f(\zeta)\,d\zeta = 0,$$

then the original value of $\dot{x}$ is restored, when $\zeta=\zeta_1$, and hence also the original values of $\dot{\zeta}$ or $\dot{z}$ on account of the equation of energy.

Let us now take a simple case so that

$$\begin{array}{llll} f(\zeta) = 1 & \text{from} & 0 & \text{to } d, \\ f(\zeta) = 0 & \text{,,} & d & \text{,, } d+l, \\ f(\zeta) = -1 & \text{,,} & d+l & \text{,, } 2d+l, \\ f(\zeta) = 0 & \text{,,} & 2d+l & \text{,, } 2d+2l, \end{array}$$

and thereafter let the form of disturbance recur.

From $\zeta = 0$ to $\zeta = d$, we have

$$\ddot{x} = \frac{eX_0}{mV}\dot{\zeta}, \qquad \ddot{\zeta} = -\frac{eX_0}{mV}\dot{x}.$$

Hence

$$\dot{x} = a+\frac{eX_0}{mV}\zeta, \qquad \dot{\zeta} = c-\frac{eX_0}{mV}x;$$

and

$$\dot{x}^2+\dot{\zeta}^2 = a^2+c^2,$$

where a and c are the initial values of $\dot{x}$ and $\dot{\zeta}$.

We also get

$$\left(a+\frac{eX_0}{mV}\zeta\right)^2+\left(c-\frac{eX_0}{mV}x\right)^2 = c^2+a^2.$$

Thus during the first pulse the ion describes a circle centre

$\left(\frac{mV}{eX_0}c, \; -\frac{mV}{eX_0}a\right)$ and radius $\frac{mV}{eX_0}(a^2+c^2)^{\frac{1}{2}}$, with angular velocity $\frac{eX_0}{mV}$.
It will leave the first pulse with velocity given by

$$\dot{x} = a + \frac{eX_0}{mV}d, \qquad \dot{\zeta} = \left\{c^2 + a^2 - \left(a + \frac{eX_0}{mV}d\right)^2\right\}^{\frac{1}{2}},$$

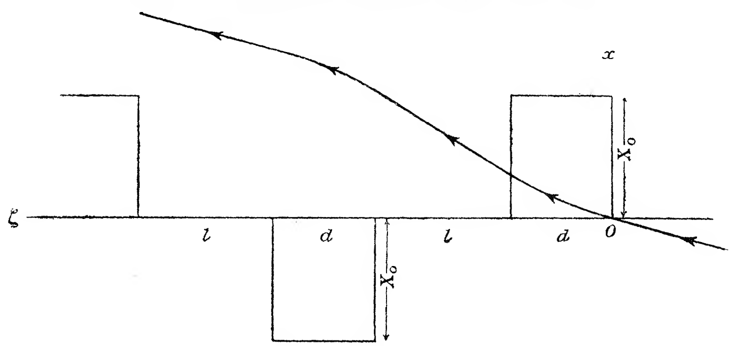
and describe a straight line until it reaches the second pulse.

During the passage through the second pulse we have

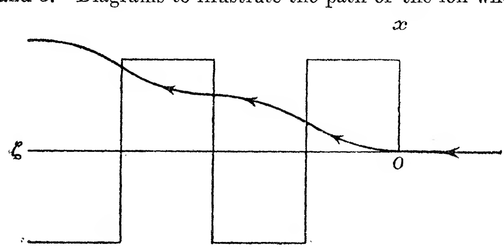
$$\ddot{x} = -\frac{eX_0}{mV}\dot{\zeta}, \qquad \ddot{\zeta} = +\frac{eX_0}{mV}\dot{x};$$

so that
$$\dot{x}^2 + \dot{\zeta}^2 = a^2 + c^2.$$

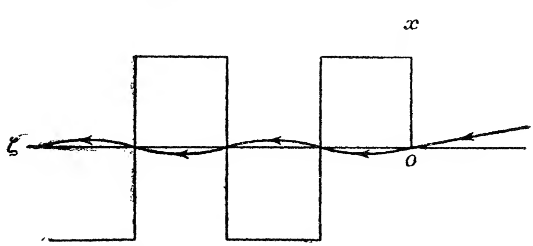
FIG. 1.—Diagram to illustrate the path of the ion.



FIGS. 2 and 3.—Diagrams to illustrate the path of the ion when $l = 0$.



Case I. $a = 0$. $c = V$.



Case II. $a = -\frac{1}{2}\omega d$. $c = V$.

Thus the ion will describe a circle of the same radius as before with the same angular velocity, but the curvature is now in the opposite direction and the centre is different. The ion leaves the second pulse with velocity (a, c), and proceeds in a straight line. The path is shown in the diagram (1).

We now calculate the time taken to travel from $\zeta = 0$ to $\zeta = 2l + 2d$, and also the displacement. The simple character of the path enables us to do this easily.

If we denote eX_0/mV by ω, the time, T, is given by

$$T = \frac{2}{\omega}\left\{\sin^{-1}\frac{a+\omega d}{(c^2+a^2)^{\frac{1}{2}}} - \sin^{-1}\frac{a}{(c^2+a^2)^{\frac{1}{2}}}\right\}$$

$$+\frac{l}{c}+\frac{l}{\{c^2+a^2-(a+\omega d)^2\}^{\frac{1}{2}}}.$$

Now, $\zeta = Vt - z$. Hence the displacement of the ion from its original position, in the direction of the waves, is

$$z = VT - 2(l+d).$$

The displacement in the direction of x is

$$x = \frac{2}{\omega}\{c - \sqrt{c^2+a^2-(a+\omega d)^2}\}$$

$$+\frac{al}{c}+\frac{(a+\omega d)\, l}{\sqrt{c^2+a^2-(a+\omega d)^2}}.$$

Let us now consider two particular cases.

Case I.—The ion was originally at rest. Hence $a = 0$ and $c = V$. Neglecting ω^4 and higher terms, we get

$$T = 2\frac{d+l}{V}+\frac{\omega^2 d^2}{V^3}\left(\tfrac{1}{3}d+\tfrac{1}{2}l\right).$$

$$z = \frac{\omega^2 d^2}{V^2}\left(\tfrac{1}{3}d+\tfrac{1}{2}l\right).$$

$$x = \frac{\omega d}{V}(d+l)+\frac{\omega^3 d^3}{V^3}\left(\tfrac{1}{4}d+\tfrac{1}{2}l\right).$$

Thus, if the impulses recur, the ion will appear to travel in space with velocities which are approximately

$$\dot{x} = \tfrac{1}{2}\,\omega d \quad \text{and} \quad \dot{z} = \frac{\omega^2 d^2}{V}\,\frac{(d+\frac{3}{2}\,l)}{6\,(d+l)}.$$

It will thus appear to drift on in the direction of the waves.*

* These are also the mean values of the true velocities $\dot{x}$ and $\dot{z}$ during a complete pulse. I wish to emphasize the fact that if we could observe the ion just as it leaves the pulse it will again be at rest if it was initially at rest, but its position is altered. I shall therefore refer to these as apparent velocities.

Case II.—Let $\dot{x} = -\frac{1}{2}\omega d$ and $\dot{z} = 0$ initially.

Thence $a = -\frac{1}{2}\omega d$ and $c = \mathrm{V}$.

In this case

$$\mathrm{T} = 2\frac{d+l}{\mathrm{V}} - \frac{\omega^2 d^2}{\mathrm{V}^3}\tfrac{1}{6}d, \text{ q.p.}$$

$$z = -\frac{\omega^2 d^2}{\mathrm{V}^2}\tfrac{1}{6}d, \text{ q.p.}$$

$$x = 0 \text{ exactly.}$$

Thus the apparent velocities are

$$\dot{x} = 0 \text{ exactly,}$$

$$\dot{z} = -\tfrac{1}{12}\frac{\omega^2 d^2}{\mathrm{V}}\frac{d}{d+l}.$$

Thus the ion will seem to move in the direction from which the impulses come. It is worth while to note that the x velocity vanishes, and so the ion will drift backwards without altering its x co-ordinate.

In both these cases the initial circumstances are such that the ion succeeds in getting through the first pulse. It will be seen that the initial circumstances can be so chosen that it fails to do so. This, however, involves the result that at some point of the circular path $\dot{\zeta} = 0$, or in other words that the ion is moving with the velocity of the waves. Now the equations break down before this point; but the result may be held to indicate that if the ion is originally moving in the direction z with a velocity a little less than V, it may, so to speak, be picked up by the waves and carried forward with the velocity V.

These cases are sufficient to illustrate the general feature, and it may be noted that the apparent x velocity is an odd function of the charge e, while the z velocity is an even function of the charge. This last result leads us to expect that even a neutral molecule made up of positive and negative ions will also be made to drift in the direction in which the waves are travelling.

We thus arrive at the conclusion that the propagation of plane polarised disturbances through a portion of space containing ions involves drifting of both positive and negative ions which may be with or against the direction of propagation according to the initial circumstances. Since the z motion does not depend on the orientation of the plane of polarisation, similar results must follow for unpolarised disturbances.

The restoration of the initial velocities relative to the fixed origin, after the passage of what we may call a complete pulse, shows that no energy (relative to the fixed origin) is permanently abstracted by the ions, although during one portion of the pulse energy is abstracted

which is exactly restored during the remaining portion. If, however, we take account of radiation from the ion, this will no longer be the case. Energy will be definitely abstracted from the pulses and radiated away from the ion. In this case the passage of a complete pulse no longer restores the original velocities unless the energy absorbed by the ions is radiated away before the pulse has passed. This will not, in general, be the case. Hence, even if radiation be taken into account, there must still be a drifting of the ions. Indeed, the general effect of the radiation will be to give the ions real velocities instead of what I have called apparent velocities.

These results seem to me to have an important bearing on the theory of Röntgen rays and of the action of radio-active substances. We may regard a radio-active substance as the origin of electro-magnetic disturbances radiated outwards. These may ionise the gas in the immediate vicinity of the substance, and we shall then have a streaming of positive and negative ions and probably also of neutral molecules, both outwards from the substance and inwards to it. This view is quite in agreement with the apparently material character of part of the radiations (indeed it would explain it), but it does not require the supposition that there is a continual diminution of the radio-active substance.

The question arises whether the velocities set up in the ions are of the order that experiment indicates. If the impulses radiated are set up by collisions of ions in the active substance, it appears to me that at least in the immediate vicinity of the substance, the velocities set up may be comparable with the velocities of the ions which produced the impulses.

The velocity of the material particles in the radiations from active substances are comparable with V. It will thus be seen that the theory suggested here requires that ωd should be comparable with V. Now ω is the angular velocity with which the ion described the circular path in passing through the pulse, and is thus the measure of the frequency of the vibrations set up in the ion. If d is of the order of a wave length of visible or ultra-violet light, then ω must be of the order of the frequency of visible or ultra-violet vibrations. Hence the theory requires that associated with the impulses we should have visible or ultra-violet light. I think it must be admitted that this is in harmony with the experimental evidence on ionising agents generally. Per contra we may argue that if any system is an origin from which electro-magnetic pulses of great intensity are radiated, we shall have associated with these, in its immediate vicinity, streams of ions moving with great velocities, and trains of waves which may be of such frequency as to come within the visible spectrum. Thus the distinction between bodies turns on the character of the impulses, and is a difference of degree rather than of kind.

Another interesting point is raised by these results. In free space the propagation of waves in a straight line is quite independent of any statical electric or magnetic field. But if the waves are propagated through a part of space containing matter, the streaming of the ions produced by the waves seems to lead to the conclusion that the propagation of the waves is no longer independent of the statical electric and magnetic field, and aberration must result.

In conclusion, I wish to express my great obligation to Professor Gray, who has discussed these results with me and read the paper with very great care.

[*Note added January* 30.—Lord Kelvin has expressed the view that a radio-active body may in some way extract energy from the æther and again radiate it. Professor and Madame Curie have also suggested a possible abstraction from the surrounding gas. The results obtained here support such views and indicate in some measure how such a process of selection may go on.]

"On the Ultra-Violet Spectrum of Gadolinium."* By Sir WILLIAM CROOKES, D.Sc., F.R.S. Received December 8, — Read December 15, 1904.

Gadolinium oxide is a rare earth, occurring between yttrium and samarium. It was discovered in 1880 by Marignac, and was at first called by him Yα, a designation which he soon changed for gadolinium. Since Marignac's time much work has been done on this earth by Lecoq de Boisbaudran, Bettendorf, Cleve, Benedicks, Marc, Demarçay, Exner and Haschek, Urbain, and others.

In the spring of this year, M. G. Urbain gave me some gadolinia and other rare earths, which he had prepared in a state of considerable purity by means of a novel system of fractionation in which use is made of the crystallisation of double nitrates of bismuth and magnesium with the rare earth nitrates. He finds that bismuth places itself between the ceric and the terbic groups, thus sharply separating samarium, the last member of the ceric group, from europium and gadolinium, the first members of the terbic groups. I have for some time past been fractionating rare earths by Urbain's method, and can quite corroborate what he says.

The ultra-violet spectrum of gadolinium has been measured by Exner and Haschek, who have published their results in a book,

[* A plate of the spectrum to which this communication refers will, it is hoped, be published in another place.]